Pearson Edexcel

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 2

Algebra and Functions (Test 2)

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. $$f(x) = 2x^3 - 5x^2 + ax + a.$$

Given that $(x + 2)$ is a factor of f (x), find the value of the constant a.

(Total for Question 1 is 3 marks)

$x + 2 = 0$

$x = -2$

$f(-2) = 2(-2)^3 - 5(-2)^2 + a(-2) + a$

$0 = -16 - 20 - 2a + a$

$0 = -36 - a.$

$a = -36$

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2. $$g(x) = \frac{2x+5}{x-3}, \quad x \geq 5.$$

(a) Find gg(5).

$g(5) = \frac{15}{2}$

(2)

$gg(5) = \frac{2\left(\frac{15}{2}\right)+5}{\frac{15}{2}-3} = \frac{20}{\frac{9}{2}} = \frac{40}{9}.$

(b) State the range of g.

$2 < g(x) \leq \frac{15}{2}$

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

$y = \frac{2x+5}{x-3}.$

swap

$x = \frac{2y+5}{y-3}$

$x(y-3) = 2y+5$

$xy - 3x = 2y + 5$

$xy - 2y = 5 + 3x$

$y(x-2) = 5 + 3x$

$y = \frac{5+3x}{x-2}$

$\therefore g^{-1}(x) = \frac{5+3x}{x-2}$

Domain:

$2 < x \leq \frac{15}{2}$

(Total for Question 2 is 6 marks)

3.

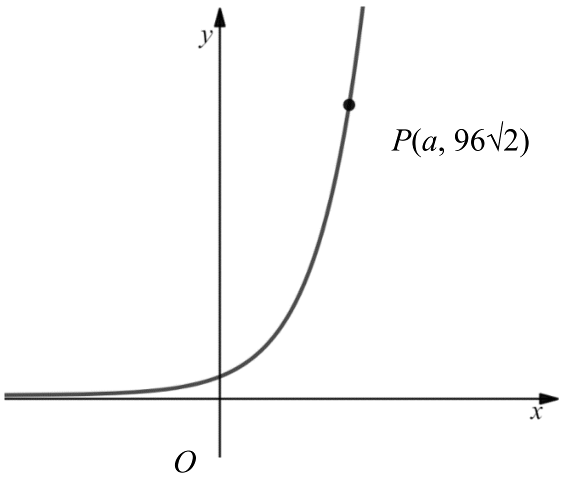


Figure 6

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 6 shows a sketch of part of the curve with equation

$$y = 3 \times 2^{2x}.$$

The point $P(a, 96\sqrt{2})$ lies on the curve.

Find the exact value of a.

$96\sqrt{2} = 3 \times 2^{2x}$

$32\sqrt{2} = 2^{2x}$

$\ln 32\sqrt{2} = 2x \ln 2.$

$\frac{11}{2} = 2x$

$\frac{11}{4} = 2.75 = x$

$x = a \quad \therefore \quad a = 2.75$

(3)

(Total for Question 3 is 3 marks)

4. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P

Find, using algebra, the exact x coordinate of P.

$$3 \times 2^x = 15 - 2^{x+1}$$

$$2^x = 5 - \frac{1}{3} 2^{x+1}$$

$$2^x = 5 - \frac{1}{3} 2^x 2^1$$

$$2^x + \frac{1}{3} 2^x 2^1 = 5.$$

$$2^x \left(1 + \frac{2}{3}\right) = 5$$

$$2^x \left(\frac{5}{3}\right) = 5$$

$$2^x = 3$$

$$x \ln 2 = \ln 3$$

$$x = 1.584962501$$

$$y = 3 \times 2^{1.58\ldots}$$

$$y = 3 \times 3$$

$$y = 9$$

$$P = (1.58, 9)$$

(4)

(Total for Question 4 is 4 marks)

5. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12 ● $(x + 4)$ is a factor of $\mathrm{f}(x)$

① FIND f(x)

$f(x) = \int 6x^2 + ax - 23.$

$f(x) = 2x^3 + \frac{a}{2}x^2 - 23x + c$

② FIND A:

$f(x) = 2x^3 + \frac{a}{2}x^2 - 23x - 12 \quad f(-4) = 0 = -128 + \frac{a}{2}16 + 92 - 12$

$0 = -128 + 8a + 80$

$0 = -48 + 8a \rightarrow 48 = 8a$

$a = 6$

③ find, in simplest form, $\mathrm{f}(x)$

SIMPUFY

	$2x^2$	$-5x$	-3
x	$2x^3$	$-5x^2$	$-3x$
$+4$	$+8x^2$	$-20x$	$+12$

$\rightarrow f(x) = 2x^3 + 3x^2 - 23x - 12$

$f(x) = (x+4)(2x^2 - 5x - 3)$

$f(x) = (x+4)(x-3)(2x+1)$

(6)

(Total for Question 5 is 6 marks)

6. $$f(x) = -3x^3 + 8x^2 - 9x + 10, x \in \mathbb{R}.$$

(a) (i) Calculate f(2).

$f(2) = -3(2^3) + 8(2^2) - 9(2) + 10 = -3(8) + 8(4) - 18 + 10$

$= -24 + 32 - 18 + 10 = 0$

(ii) Write f(x) as a product of two algebraic factors.

	$-3x^2$	$2x$	-5
x	$-3x^3$	$2x^2$	$-5x$
-2	$+6x^2$	$-4x$	$+10$

$\therefore f(x) =$
$(x-2)(-3x^2+2x-5)$

(3)

Using the answer to part (a) (ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0,$$

let $x = y^2$

$\Rightarrow -3x^3 + 8x^2 - 9x + 10 = 0$

$(x-2)(-3x^2+2x-5) = 0$

$x - 2 = 0$

$x = 2$

$-3x^2+2x-5 = 0$, $b^2 - 4ac = 4 - 60 = -56$

$\sqrt{56}$ is not real

$x = y^2 = 2$

$y = \pm\sqrt{2}$.

(2)

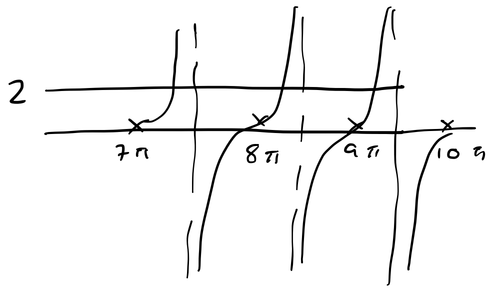
(c) deduce the number of real solutions, for $7\pi \leq \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0.$$

(1)

$\tan\theta = 2$

$\theta = \tan^{-1} 2 = 1.1$ radians



$\therefore$ there will be 3 roots

(Total for Question 6 is 6 marks)

$$2(x+1)(x+1)$$
$$= 2(x^2+2x+1)$$

7. $\mathrm{f}(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$

$$= 2x^2 + 4x - 2 + 9$$
$$= 2x^2 + 4x + 7$$

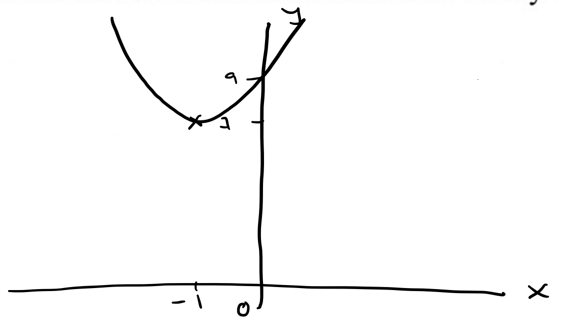
(*a*) Write f (*x*) in the form $a(x + b)^2 + c$, where *a*, *b* and *c* are integers to be found.

$2(x+1)^2 + 7 \qquad a = 2,\ b = 1,\ c = 7$

(3)

(*b*) Sketch the curve with equation $y = \mathrm{f}(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)



Turning point $(-1, 7)$

$2(x+1)^2 = 0$

$\therefore x = -1$

$\therefore y = 7$

Points of intersection

y-intercept $= 9 \quad (0, 9)$

x-intercept $=$ none.

(3)

(*c*) (i) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = 2(x-2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

$g(x) = f(x-2) + k$

$= 2(x-2)^2 + 4(x-2) + 9 + k$

$= 2(x-2)^2 + 4x - 8 + 9 + k$

$-8 + 9 + k = -3$

$1 + k = -3$

$k = -4.$

$\therefore g(x) = f(x-2) - 4$, translation 2 to the right and 4 down

$\therefore$ translation $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(ii)Find the range of the function

$$\mathrm{h}(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

$h(x) = \dfrac{21}{f(x)} = \dfrac{21}{2(x+1)^2 + 7}$

using $f(x)$ turning point:

$h(-1) = \dfrac{21}{7} = 3$

HORIZONTAL ASYMPTOE

$\lim_{x \to \infty} \left(\dfrac{21}{2x^2 + 4x + 9} \right) = 21 \lim_{x \to \infty} \left(\dfrac{1}{2x^2 + 4x + 9} \right)$

$= 21 \dfrac{1}{\infty} = 21(0) \therefore y = 0$ is the horizontal asymptote.

$\therefore$ range is

$0 < h \leq 3$

(Total for Question 7 is 10 marks)

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8.
$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}.$$

(a) Find the values of the constants A, B and C.

$1+11x-6x^2 = A(x-3)(1-2x) + B(1-2x) + C(x-3).$

(4)

FOR $x = \frac{1}{2}$: $1 + \frac{11}{2} - \frac{3}{2} = C(-\frac{5}{2}) \longrightarrow 5 = -C\frac{5}{2}$

$-2 = C$

FOR $x = 3$ $\quad 1 + 33 - 54 = -20 = B(1-6)$

$-20 = B(-5)$

$\frac{20}{5} = 4 = B$

FOR A:

$-6x^2 = A(x - 2x^2 - 3 + 6x)$

$= A(-2x^2 + 7x - 3)$

$\therefore -6x^2 = -A2x^2 \longrightarrow 6 = 2A$

$A = 3$

$$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)}, \quad x > 3.$$

$A = 3$

$B = 4$

$C = -2$

(b) Prove that f (x) is a decreasing function.

$f(x) = 3 + \frac{4}{x-3} + \frac{-2}{1-2x}$

(3)

$= 3 + 4(x-3)^{-1} - 2(1-2x)^{-1}$

$\therefore f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$

$f'(3) = \frac{-4}{25} \longrightarrow f'(3) < 0$, $\therefore$ it is a decreasing function. for $x > 3$

(Total for Question 8 is 7 marks)

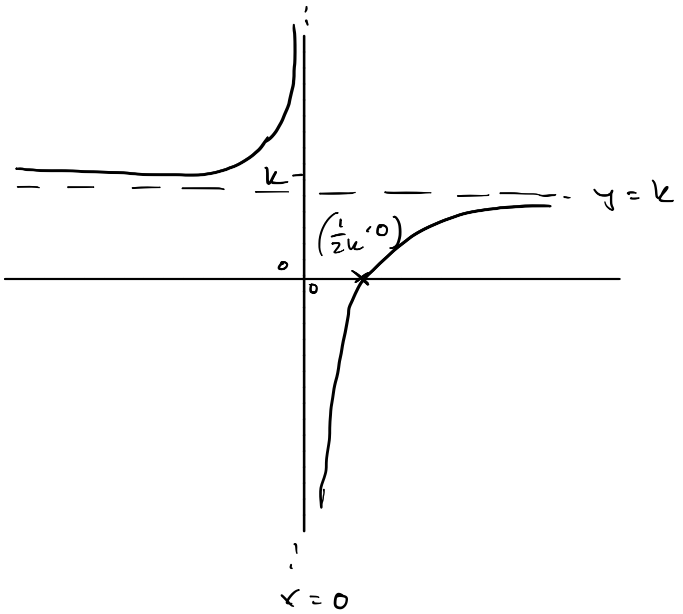
9. (a) Sketch the curve with equation

$$y = k - \frac{1}{2x},$$

where k is a positive constant.

State, in terms of k, the coordinates of any points of intersection with the coordinate axes and the equation of the horizontal asymptote.

(3)



POINTS OF INTERSECTION

x - intercept

$0 = k - \frac{1}{2x} \rightarrow \frac{1}{2x} = k$ $\quad \left(\frac{1}{2k}, 0\right)$

$1 = 2kx$

$\frac{1}{2k} = x$

y - intercept

$y = k - \frac{1}{0} \rightarrow$ no intercept asymptote at $x = 0$

HORIZONTAL ASYMPTOTE

$k - \lim_{x \to \infty} \frac{1}{2x} = k - \frac{1}{\infty} = k - 0 = k$

$y = k$ is the horizontal asympotel

The straight line l has equation $y = 2x + 3$.

Given that l cuts the curve in two distinct places,

(b) find the range of values of k, writing your answer in set notation.

(6)

(Total for Question 9 is 9 marks)

l: $y = 2x + 3$

$2x + 3 = k - \frac{1}{2x}$

$2x(2x) + 3(2x) = k(2x) - 1$

$4x^2 + 6x = 2kx - 1$

$4x^2 + (6 - 2k)x + 1 = 0$

$a = 4$

$b = 6 - 2k$

$c = 1$

Two distinct solutions

$b^2 - 4ac > 0$

$(6 - 2k)^2 - 4(4)(1) > 0$

$(6 - 2k)(6 - 2k) - 16 > 0$

$36 - 24k + 4k^2 - 16 > 0$

$4k^2 - 24k + 20 > 0$

$k^2 - 6k + 5 > 0$

$(k - 5)(k - 1) > 0$

$k - 5 > 0 \quad k - 1 > 0$

$k > 5 \quad k > 1$

$(k > 5) \cap (k > 1)$

10.

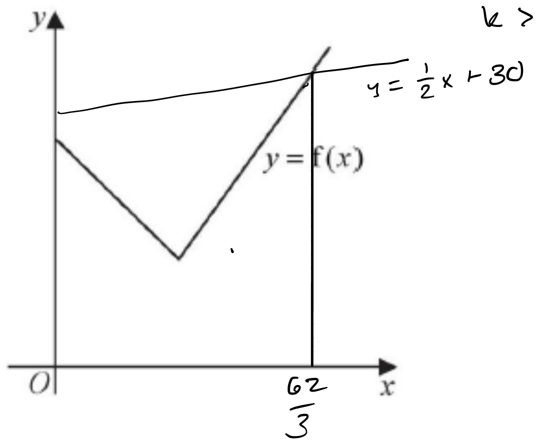


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$ where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(*a*) State the range of f.

$y \geq 5$.

(1)

(*b*) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

$2|3 - x| + 5 = \frac{1}{2}x + 30$

$2|3 - x| = \frac{1}{2}x + 25$

$|3 - x| = \frac{1}{4}x + \frac{25}{2}$

Solution 1

$3 - x = \frac{1}{4}x + \frac{25}{2}$

$-\frac{19}{2} = \frac{5}{4}x$

$x = -7.6$, however $x \geq 0$ so this is invalid

Solution 2

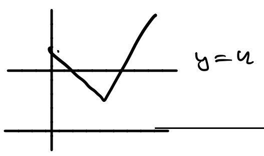
$-3 + x = \frac{1}{4}x + \frac{25}{2}$

$\frac{3}{4}x = \frac{31}{2} \rightarrow x = \frac{62}{3} = 20.7$ (3sf)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(*c*) state the set of possible values for k.

(2)



$5 < k \leq 11$

as after $k = 11$ there will only be one solution.

(Total for Question 10 is 6 marks)

11.

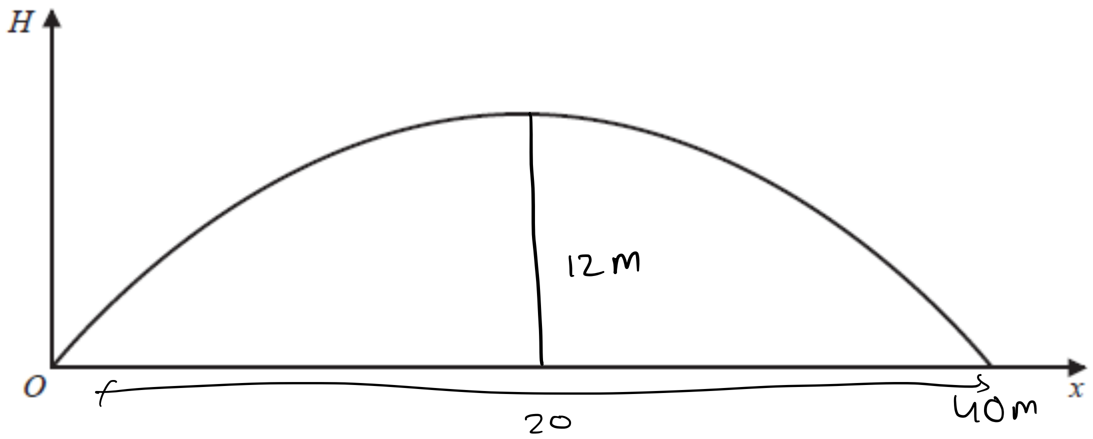


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

$H = kx(x - 40)$ **(3)**

$12 = k(20)(20 - 40) = -400$

$k = \frac{12}{-400} = \frac{3}{-100} = -0.03$ $\therefore$ $H = -0.03x(x - 40)$

$H = -0.03x^2 + 1.2x$

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

$3 = -0.03x^2 + 1.2x$

$0.03x^2 - 1.2x + 3 = 0$

$3x^2 - 120x + 300 = 0$

$x_1 = 20 + 10\sqrt{3} = 37.3205$

$x_2 = 20 - 10\sqrt{3} = 2.679491924$

$x_1 > x_2$ **(3)**

$\therefore$ greatest horizontal distance $= 37.3$ m (3sf)

(c) Give one limitation of the model.

It does not take air resistance into account. **(1)**

(Total for Question 11 is 7 marks)